



## **RESEARCH DEPARTMENT**

### **A METHOD OF AMPLITUDE AND PHASE MEASUREMENT IN THE V.H.F./U.H.F. BAND**

**Report No. E-067**

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**THE BRITISH BROADCASTING CORPORATION  
ENGINEERING DIVISION**

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IN THE V.H.F./U.H.F. BAND

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K.W.T. Hughes, B.Sc.  
G.D. Monteath, B.Sc., A.Inst.P., A.M.I.E.E.  
D.J. Whythe, B.Sc.(Eng.), A.M.I.E.E.

W. Proctor Wilson

(W. Proctor Wilson)

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## A METHOD OF AMPLITUDE AND PHASE MEASUREMENT IN THE V.H.F./U.H.F. BAND

### SUMMARY

A null method is described for measuring changes in the amplitude and phase of the transmission characteristic of a network at any frequency in the range 41 Mc/s to 1000 Mc/s. A commercial instrument, designed for admittance measurement, is used with only slight modification for this application.

The measurement of phase can be made to within about  $\pm 3^\circ$  unless a large variation of amplitude is encountered. Reading accuracy then limits the accuracy of phase measurement where the amplitude is smaller. If the error in a measurement is regarded as a vector, the maximum value of the magnitude of this vector is about 6% of the full-scale reading of the instrument.

### 1. INTRODUCTION

Several methods have been devised for measuring the phase difference between two alternating voltages at high frequencies. One phase meter for microwave frequencies<sup>1</sup> employs a balanced modulator in conjunction with a calibrated phase shifter in a waveguide section. In another system<sup>2</sup>, of use in the v.h.f. and u.h.f. bands, the output voltage from the network under test is balanced against a subsidiary voltage obtained from the probe of a well matched slotted line. This subsidiary voltage is adjusted in amplitude by an attenuator, and in phase by traversing the probe along the slotted line. In a third method<sup>3</sup>, which has been used to compare the outputs of directional couplers, two signals are compared at an intermediate frequency. Two outputs of a local oscillator are fed with an adjustable phase difference to two frequency changers. This phase difference is adjusted to give intermediate frequencies which are in antiphase, and their amplitudes are compared with a potentiometer.

This report describes a new method for measuring relative phase at any frequency in the range 41 Mc/s to 1000 Mc/s, using an instrument designed primarily for admittance measurement<sup>4,5</sup>.

### 2. METHOD OF MEASUREMENT

The output from an oscillator is fed to a receiver by two parallel paths. One path includes the network under test and the other includes an instrument whose output can be adjusted in amplitude and phase to cancel the output from the network. This arrangement is shown in Fig. 1.

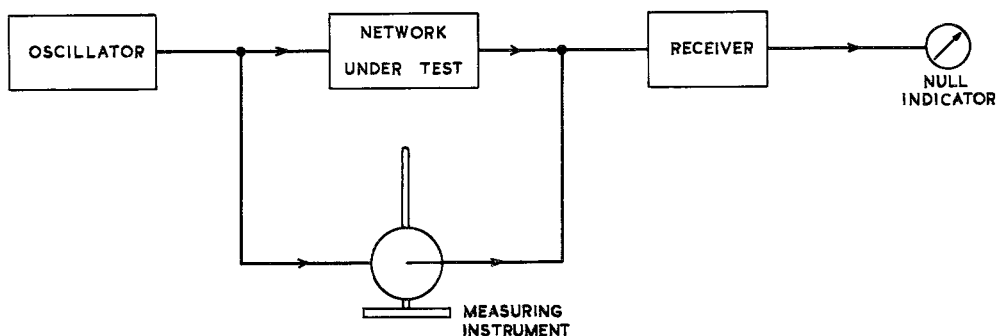


Fig. 1 - Principle of measurement

The instrument used for controlling the amplitude and phase of the cancelling voltage is the General Radio admittance meter<sup>4, 5</sup>. A schematic diagram reproduced from the makers' instructions is shown in Fig. 2. Three coaxial transmission lines have a common junction connected to the input of the meter. For admittance measurement these lines are terminated respectively by a conductance standard, a susceptance standard and the unknown admittance. The conductance and susceptance of the standards are numerically equal to the characteristic admittance of the lines, the susceptance standard consisting either of an adjustable short-circuited transmission line or a variable capacitor. The current entering each line is sampled by a loop which can be rotated to vary its coupling; the outputs from the loops are combined and a null is obtained when the sum of the voltages induced in

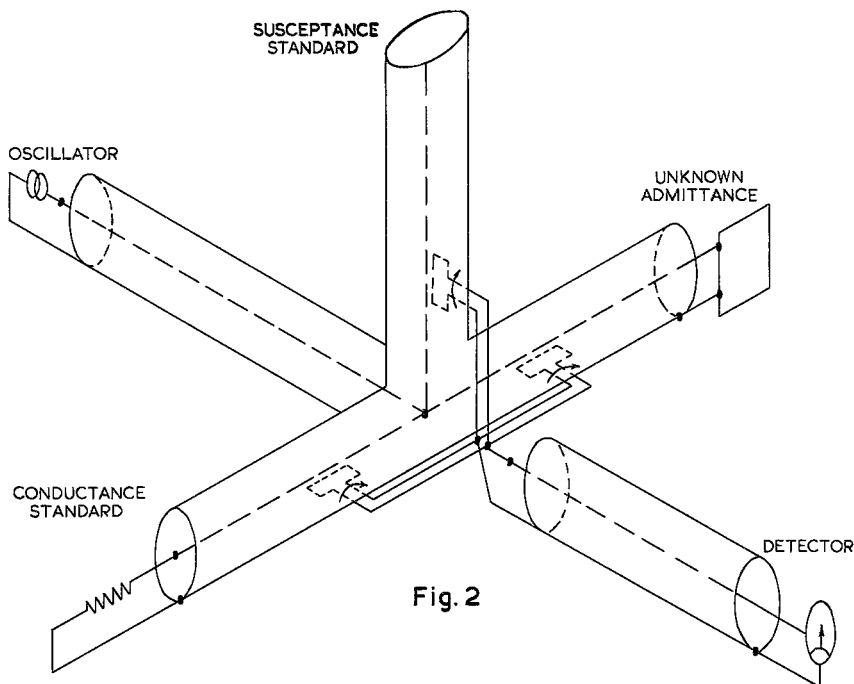


Fig. 2 - General arrangement of the admittance meter  
(reproduced from the makers' instructions)

them is zero. Scales associated with the three loops enable the admittance to be read directly in terms of conductance in millimhos, susceptance in millimhos and a scale-multiplying factor.

When the admittance meter is used in the circuit of Fig. 1 to measure amplitude and phase, the scale-multiplying factor is set to infinity, so that there is no coupling between the "unknown" transmission line and the corresponding loop. The output then has two components in phase quadrature; their amplitudes are proportional to the readings of the "susceptance" and "conductance" scales respectively. When these scales are set for zero output in the receiver, they indicate the real and imaginary parts of the complex output of the network under test, relative to some arbitrary standard.

The loop coupled to the conductance standard can be rotated only through  $90^\circ$ , so that use of the "conductance" and "susceptance" scales can give only a  $180^\circ$  phase variation. In order to cover  $360^\circ$  an additional standard conductance is plugged in to the "unknown" connector. The loop coupled to this gives an output in antiphase to that obtained from the "conductance" loop. The use of a second standard conductance, though convenient, is not essential, since a single standard conductance can be moved from one position to the other according to the sign of the real part required. The unused loop is set to zero coupling (zero mmhos or infinity scale factor).

As supplied by the makers, the admittance meter is designed for use with 50-ohm transmission line, and the standard conductance and susceptance values are therefore 20 mmhos. If the "conductance", "susceptance" and "multiplier" scales respectively indicate  $G$  mmhos,  $B$  mmhos and  $M$ , and the unknown arm is terminated in  $Y_0$  [ $Y_0 = (20 + j0)$  mmhos]; the output is equal to  $K(G - 20/M + jB)$  where  $K$  is a complex constant. It is convenient to superimpose on the "multiplier" scale a detachable scale with engravings similar to those of the "conductance" scale. If this reads  $G'$  the output is proportional to  $K(G - G' + jB)$ .

### 3. EXPERIMENTAL RESULTS

Experiments were performed at 500 Mc/s and 1000 Mc/s to assess the accuracy of measurement. The method described in Section 2 was adopted to measure the relative amplitude and phase of the output voltage from the probe of a slotted line for various standing-wave ratios on the line. The results are compared with curves which were calculated from the standing-wave ratios, measured independently against a piston attenuator. At each frequency four terminations were used to give voltage standing-wave ratios between 0.98 and 0.007. A preliminary examination of the standing-wave pattern showed the effect of loss and discontinuities in the slotted line to be negligible.

It is shown in the Appendix that, in the absence of loss, the locus (on the Argand diagram) of the voltage on a uniform transmission line is an ellipse. The way in which position on the ellipse varies with position on the transmission line is illustrated in Fig. 3. A point P on a circle is defined by the angle  $\theta$ , which is equal to the angular distance along the transmission line from the voltage maximum. The point Q on the perpendicular PN to the axis is defined by  $QN/PN = s$ , where  $s$  is the voltage standing-wave ratio. If the probe is moved at a uniform rate along the slotted line, P moves at a uniform rate round the circle and Q traces out an ellipse. The circle is the auxiliary circle of the ellipse.

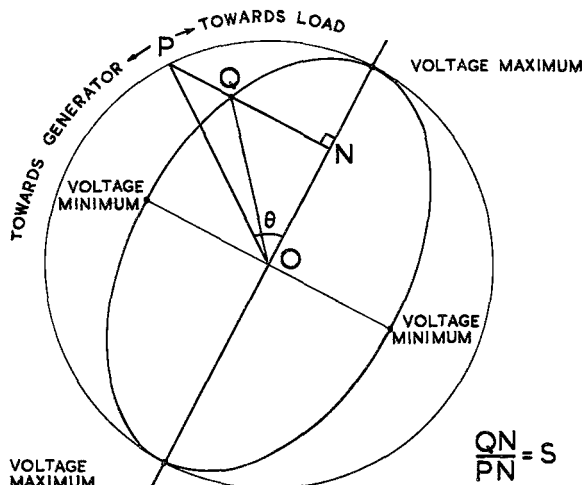


Fig. 3 - The locus of the voltage vector (OQ) on a uniform transmission line

Measurements are made with the greatest accuracy when the whole of the "conductance" and "susceptance" scales are used. This occurs when the axes of the ellipse are inclined at approximately  $45^\circ$  on the Argand diagram. To achieve this result a phase shifter (a variable length of coaxial line) was placed in series with the admittance meter and adjusted before each measurement commenced. The results of the measurements are compared in Figs. 4 and 5 with the corresponding theoretical points lying on the ellipse. For clarity, overlapping points measured with the probe in different positions on the slotted line have been differently coded. The eccentricity of each theoretical ellipse was determined from the measured standing-wave ratio on the slotted-line while the positions of the points were calculated from the measured displacement of the probe from the voltage antinode. The size and inclination of the theoretical ellipse was adjusted arbitrarily to give the best fit to the experimental points.

#### 4. MEASUREMENT ERRORS

The source of greatest error was found to be that of scale reading, particularly when the network attenuation varied over a wide range. The meter scales are marked in approximately equal intervals of amplitude, which makes accurate reading difficult at the minima. Phase measurement errors are therefore greatest where the magnitude is a minimum. The accuracy of phase measurement at 500 Mc/s and 1000 Mc/s is better than  $\pm 3^\circ$  if the range of attenuation is within 6 dB, but, owing to lack of scale discrimination, errors up to  $\pm 10^\circ$  may occur at the minima if the range is extended to 40 dB.

A convenient method of expressing the accuracy is in terms of the vector error. If  $\delta g$  and  $\delta b$  are the errors in the readings of the real and imaginary parts, obtained from the "conductance" and "susceptance" scales of the admittance meter, the magnitude of the vector error is  $(\delta g^2 + \delta b^2)^{\frac{1}{2}}$ . It corresponds to the length of a line drawn on the Argand diagram between theoretical and experimental points. If the maximum readings of the scales are  $g_m$  and  $b_m$ , then the relative vector error  $\epsilon$  may be defined as

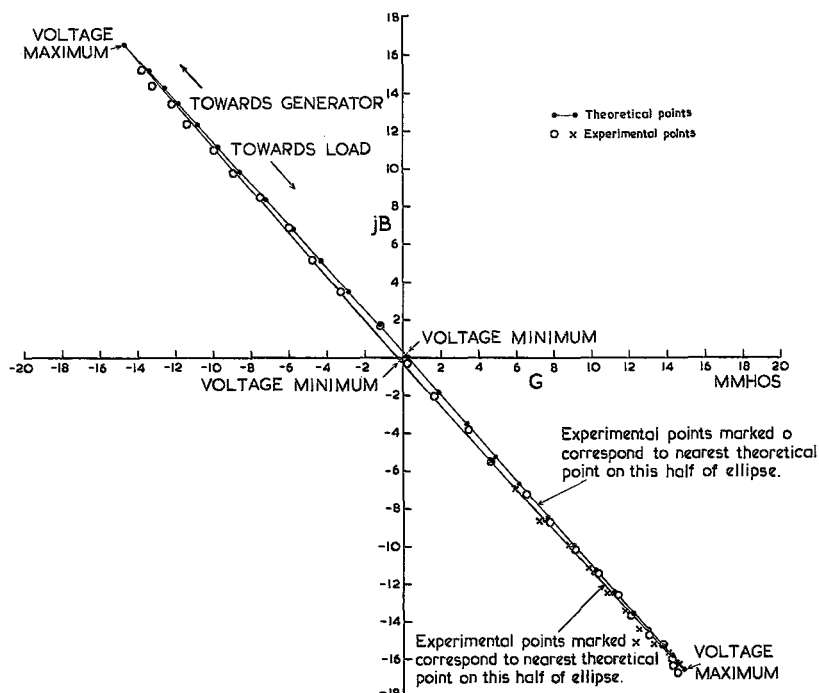


Fig. 4(a) - Complex voltage distribution along a slotted line  
SWR = 0.007;  $f = 500$  Mc/s

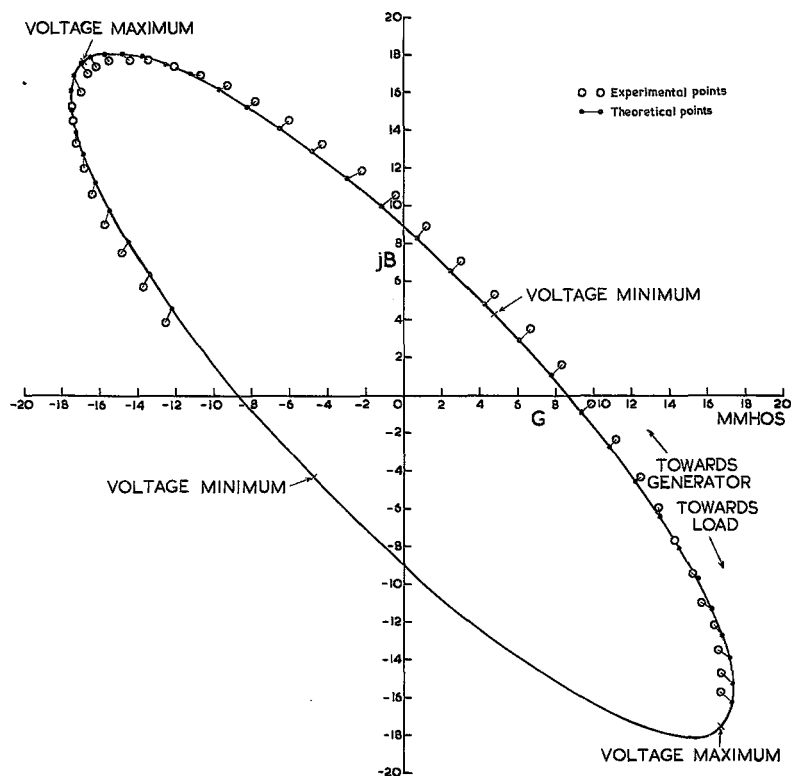


Fig. 4(b) - Complex voltage distribution along a slotted line  
SWR = 0.263;  $f = 500$  Mc/s

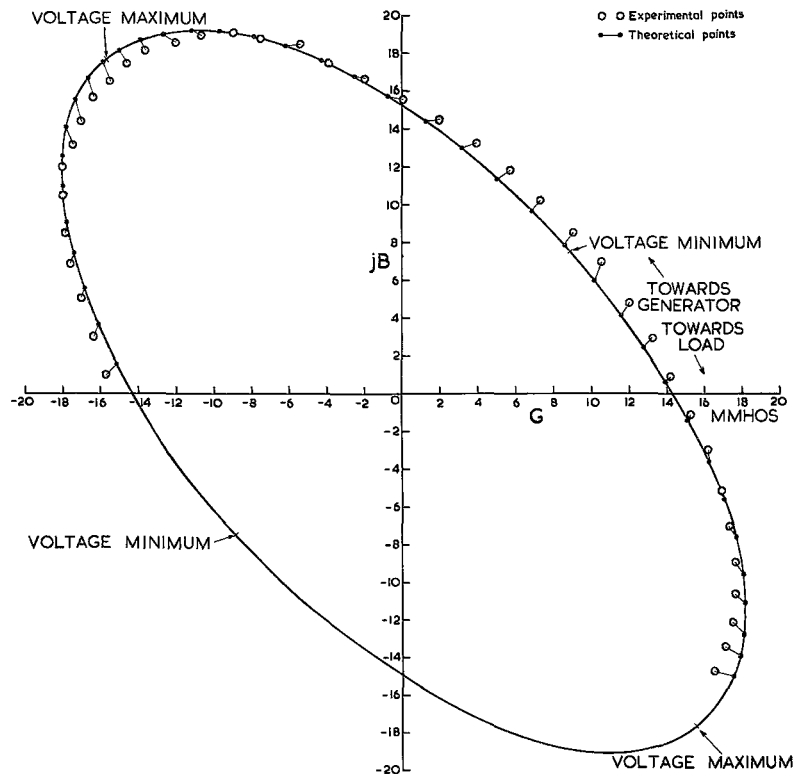


Fig. 4(c) - Complex voltage distribution along a slotted line  
 $SWR = 0.5$ ;  $f = 500$  Mc/s

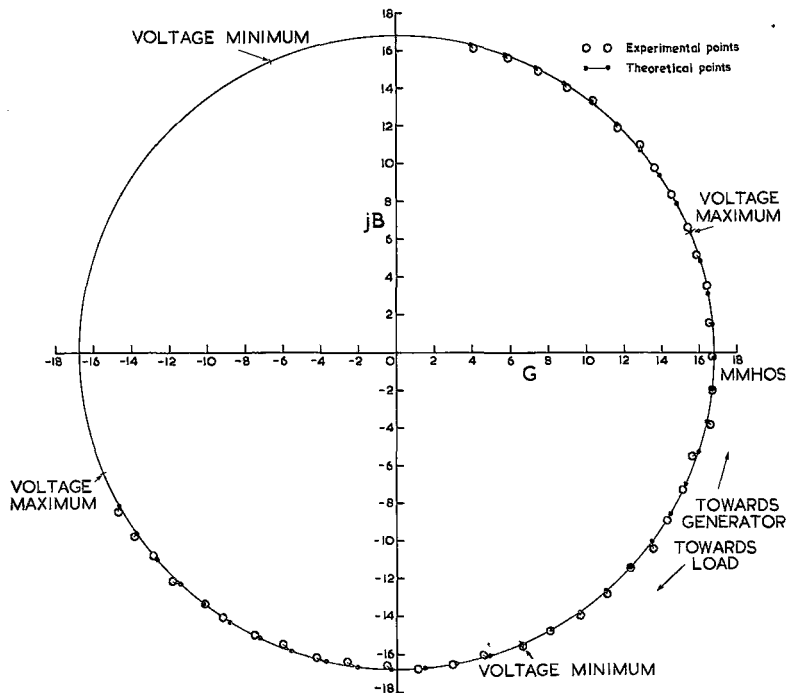
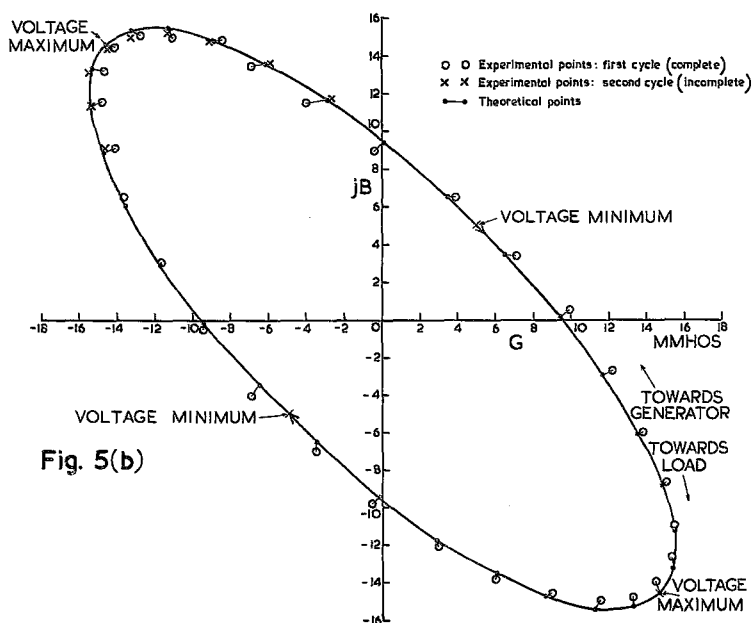
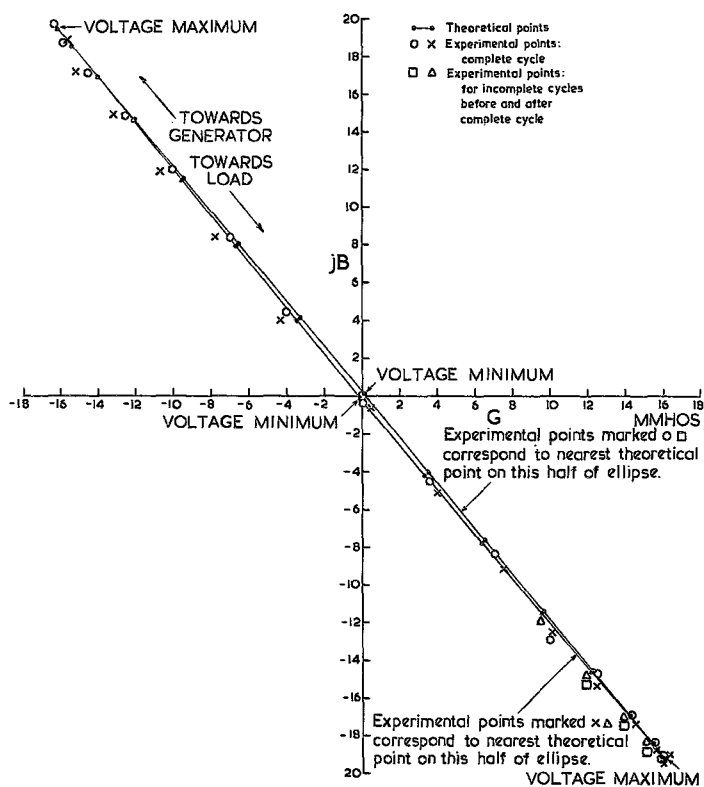


Fig. 4(d) - Complex voltage distribution along a slotted line  
 $SWR = 0.98$ ;  $f = 500$  Mc/s



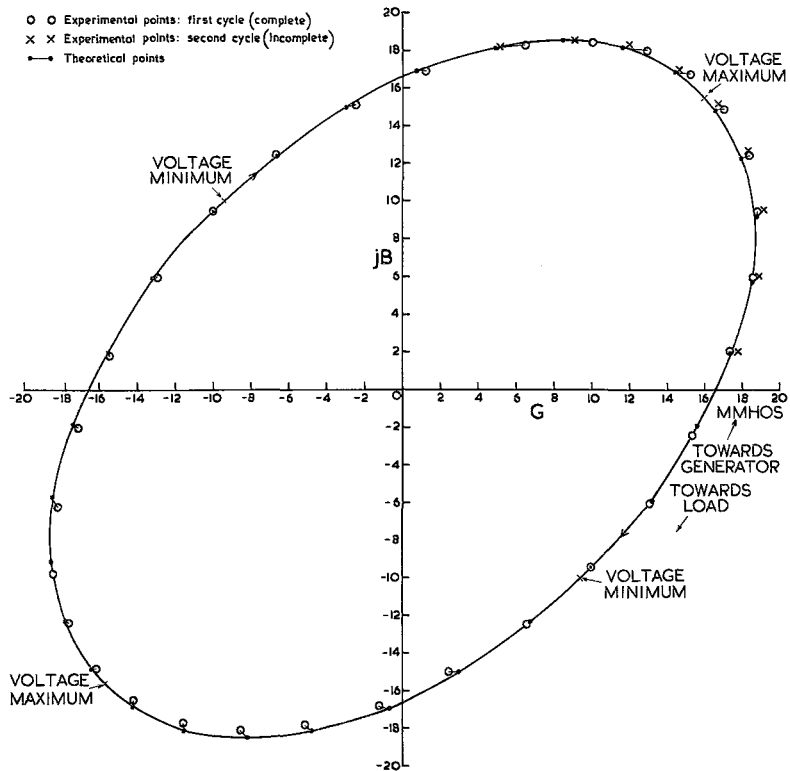


Fig. 5(c) - Complex voltage distribution along a slotted line  
 $SWR = 0.62$ ;  $f = 1000$  Mc/s

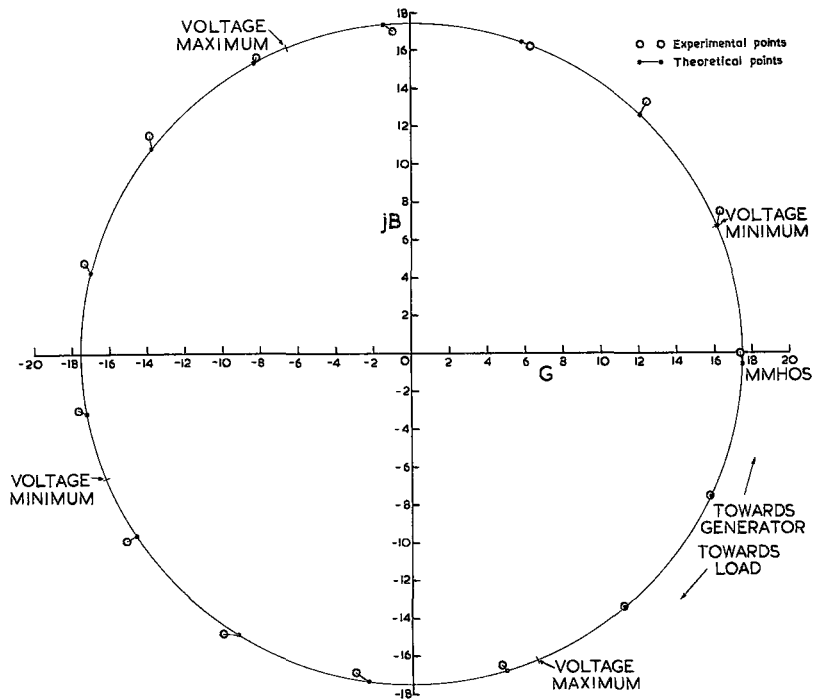


Fig. 5(d) - Complex voltage distribution along a slotted line  
 $SWR = 0.98$ ;  $f = 1000$  Mc/s

$$\epsilon = (\delta g^2 + \delta b^2)^{\frac{1}{2}} / (g_m^2 + b_m^2)^{\frac{1}{2}}$$

The maximum value of  $\epsilon$  is 6% approximately for the frequencies and standing-wave ratios used in the experiments.

It is believed that the greater part of the error between experimental and theoretical results is due to the difference (5-10%) in eccentricity between the measured and theoretical ellipses. This discrepancy may be due to either admittance meter errors or imperfect screening of the apparatus.

A large difference (many wavelengths) between the lengths of the two paths from oscillator to receiver should be avoided; otherwise the accuracy of measurement will be very dependent on the frequency stability of the signal source.

## 5. CONCLUSIONS

The method of measurement described has proved most useful in measuring the radiation patterns of model aerial systems, since information of the phase variation often makes it possible to work with a very simple model. The "network" consisted of the aerial under test, which could be rotated, a fixed aerial, and the space between them. For example, where a v.h.f. transmitting aerial consisting of rings of radiating dipoles surrounding a supporting mast is required to have a horizontal radiation pattern of a particular shape, a model mast section is arranged to carry only one dipole. The radiation pattern is then measured in amplitude and phase, and from this is calculated the radiation pattern of a complete ring, with the dipoles energised in any manner. In this way the best choice of the amplitudes and phases of the feeds to the dipoles may be determined by calculation. One advantage of this technique is that the dipole used need not be well matched to the feeder. If a complete ring of elements were used for the measurement, great care in matching would be required to achieve the required amplitudes and phases of the radiating currents.

Simultaneous phase and amplitude measurement has also proved useful for adjusting directional couplers<sup>6</sup>, where the effect of imperfect directivity has to be distinguished from that of an imperfectly-matched load.

## 6. REFERENCES

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4. General Radio Experimenter, May 1950 and August 1953.
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## APPENDIX

## THE DISTRIBUTION OF VOLTAGE ON A UNIFORM TRANSMISSION LINE

The voltage on a uniform transmission line may be regarded as the sum of two components, associated respectively with waves travelling in two directions. Neglecting loss, and assuming only one frequency to be present, the voltage  $V$  may be expressed as

$$V = V_i e^{j\beta x} + \rho V_i e^{-j\beta x} \quad (1)$$

where  $\beta x$  is the angular distance measured along the line from an arbitrarily chosen origin towards the source,  $V_i$  is the voltage associated with the incident wave at the origin, and  $\rho$  is the reflection coefficient at the origin. In general  $\rho$  will be complex.

Let

$$\rho = |\rho| e^{j\phi}$$

Then

$$\begin{aligned} V &= V_i e^{j\phi/2} \{ e^{j(\beta x - \phi/2)} + |\rho| e^{-j(\beta x - \phi/2)} \} \\ &= V_i e^{j\phi/2} [(1 + |\rho|) \cos(\beta x - \phi/2) + j(1 - |\rho|) \sin(\beta x - \phi/2)] \end{aligned} \quad (2)$$

Now the parametric equations of an ellipse centred at the origin, with the major and minor axes (lengths  $2a$  and  $2b$ ) oriented along the  $x$  and  $y$  axes are

$$\left. \begin{aligned} x &= a \cos \theta \\ y &= b \sin \theta \end{aligned} \right\} \quad (3)$$

Writing  $z = x + jy$ , the equation of the ellipse as a locus on the Argand diagram is

$$z = a \cos \theta + jb \sin \theta \quad (4)$$

Comparing equation 4 with equation 2 it is seen that if the quantity in square brackets is plotted on the Argand diagram, the locus traced is an ellipse with major and minor semi-axes  $(1 + |\rho|)$  and  $(1 - |\rho|)$  on the real and imaginary axes at the  $z$ -plane. The ratio of the axes,  $(1 - |\rho|)/(1 + |\rho|)$ , is equal to the voltage standing-wave ratio. When  $V$  is plotted this ellipse is scaled in proportion to  $V_i$  and rotated through the angle  $\phi/2$ .